MODELLING OF GROUNDWATER FLOW IN FRACTURED ROCK: THEORETICAL APPROACH AND PRACTICAL APPLICATIONS

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Abstract
The poster gives an overview of the present knowledge achieved by our research group in the field of modelling of the fluid flow in a fractured rock environment. First, we introduce possible approaches. Then we describe a process of generation of a mesh. Our system is based on the discrete stochastic network approach. It means that particular fractures are represented as 2D entities (polygons in our case) placed in 3D space. The resulting mesh has the same statistical characteristic (density of fractures, their orientation, permeability etc.) as real fractured environment in the rock massif. The second part of the poster describes a numerical model of the groundwater flow. This model solves the problem using linear (Darcy) flow. A Mixed-hybrid FEM is used for approximation of the PDE's. In the third part we show application of our method to a real-world hydrogeological problem - simulation of the injection test and communication between drillholes PTP3 and PTP4a in the Krušné Hory Mountains in the Czech Republic.

Additional Keywords: fractured rock, numerical modelling, Darcy law, and finite element method.

Introduction
Numerical modelling of processes in the fractured rock environment becomes an important tool for solving many hydrogeological, geochemical and ecological problems. The most important practical application is expected in choosing the most suitable locality for the permanent repositories of dangerous (especially radioactive) waste. This kind of modelling is a relatively new branch of research. A lot of software exists for modelling processes in fractured rock, but no of them is considered to be generally applicable for all possible types of problems.

The start of the construction of the radioactive waste repository in the Czech Republic is planned in the time horizon of approximately twenty years but some preparation works have already started. In the scope of this works, a team of researchers from the Czech Geological Survey, Technical University of Liberec and Institute of Computer Science of the Academy of Sciences of the Czech Republic has been established. The goal of this team is to develop a software system for the simulation of flow processes in fractured rock, with special respect to geological conditions typical for the crystalinic massifs in the Czech Republic. There were two grant projects (GAČR 205/00/0480 and MŽP VaV 630/3/00) focused on this area. As a result of these projects, there exists the first functional and applicable version of the simulation system.

Stochastic Discrete Fracture Network Approach
Underground granitoid massifs are proposed as nuclear waste repositories. However, they are always disrupted by a system of geological faults and fractures. The percolation of groundwater through such massifs is called *fracture flow*.
According to Bear (1993) there are three main approaches to modeling of fracture flow:
1. *Equivalent porous medium models* are used for large-scale problems if there is no need for knowing the flow field in detail.
2. *Double porosity models* work with two connected continua – representing fractures and porous blocks.
3. *Stochastic discrete fracture network models* are trying to create as exact representation of the fractured environment as possible by simulating particular fractures. Due to computational costs these models can be used only for solution of the problems on relatively small domains (up to tens of meter).
4.

Our simulation system is based on the third approach. The main idea of that approach is to approximate the original three-dimensional fractures by planar elliptic or polygonal disks whose frequency, size, assigned aperture, and orientation are statistically derived from field measurements and consider two-dimensional Darcy flow in such a network. Therefore the procedure of calculation of a flow field by such approach has two important steps:
1. Generate a stochastic discrete fracture network.
2. Find an approximation of the flow field on such network.

We will describe both of these topics in following two sections.

Generation of the fracture network

We generated the fracture network on the basis of statistical data obtained from field measurements. We enabled the definition of hydraulically important fractures, zones with an increased density of fractures, or insertion of deterministic fractures. Planar circle disks approximate the original three-dimensional fractures and each disk is subsequently discredited into a triangular mesh respecting the intersections with his neighbors. In order to simplify the geometrical situation in fracture planes, the computed intersections are moved and stretched slightly. In this way, one obtains a mesh of a higher quality; however, the three-dimensional geometrical correspondence vanishes and has to be replaced by an element edges correspondence. Finally, we assign an aperture to each element. Based on it, the hydraulic permeability of the element is set, considering also fracture wall roughness and filling. The classical parallel plate model is thus avoided and the channeling effect is simulated. We can see an example of a simple triangular mesh in Figure 1.

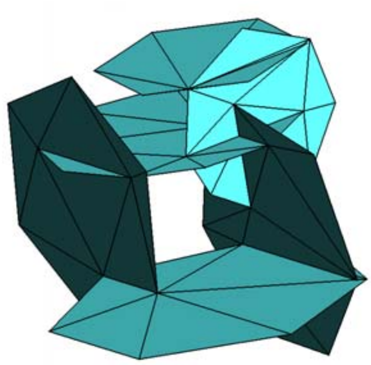

Figure 1. Fracture network made of 3 polygons, discredited into a triangular mesh

Approximation of the flow field

System of fractures S can be denoted as:

$$S \equiv \left\{ \bigcup_{l \in L} \overline{\alpha_l} \setminus \partial S \right\}, \tag{1}$$

with α_l an opened 2-D polygon placed in the 3-D Euclidean space. We call $\overline{\alpha_l}$ the closure of α_l, a *fracture. L* is the index set of fractures, ∂S is the set of those boundaries of α_l which do not create the connection with other fractures. We suppose that all α_l are connected into one *fracture network*; the connection is possible only through an edge, not a point.

We are looking for the fracture flow velocity **u** (2-D vector in each α_l), which is the solution of the problem

$$\mathbf{u} = -\mathbf{K}(\nabla p + \nabla z) \qquad \text{in } S, \tag{2}$$

$$\nabla \cdot \mathbf{u} = q \qquad \text{in } S, \tag{3}$$

$$p = p_D \quad \text{in } \Lambda_\mathrm{D}, \quad \mathbf{u} \cdot \mathbf{n} = u_N \quad \text{in } \Lambda_\mathrm{N}, \tag{4}$$

where all variables are expressed in local coordinates of appropriate α_l, and also the differentiation is always expressed in towards these local coordinates. The equation (2) is Darcy's law, (3) is the mass balance equation and (4) is the expression of appropriate boundary conditions. The variable p denotes the modified fluid pressure $p = \mathrm{p}/(g\rho)$, g is the gravitational acceleration constant, ρ is density of the fluid, q represents stationary sources/sinks density and z is the elevation, positive upward vertical 3-D coordinate expressed in local coordinates of appropriate α_l. The second rank tensor **K** of hydraulic conductivity is the function of original 3-D fracture properties mentioned above (aperture...). We require it to be symmetric and uniformly positive definite on each α_l. We pose also the requirement $\Lambda_D \cap \Lambda_N = \emptyset, \overline{\Lambda_D} \cup \overline{\Lambda_N} = \partial S, \Lambda_D \neq \emptyset$.

The detailed description of the process of the approximation of the problem described above is out of the scope of this paper. For details see for example Maryška, Severýn, Vohralík (2002). There are three main ideas of the formulation of the continuous problem. i.) Divide the whole domain S to subdomain e – called *elements* – of a simple geometrical shape. ii.) Require a weak fulfillment of the equations (2), (3), (4) on each e. iii.) Express conservation of the mass on each boundary between two elements e_i and e_j of the mesh. Mathematical expression of these ideas leads to a system of integral identities

$$\sum_{e\in\mathcal{E}_h}\{(\mathbf{A}\mathbf{u}^e,\mathbf{v}^e)_{0,e}-(p^e,\nabla\cdot\mathbf{v}^e)_{0,e}+\langle\lambda^e,\mathbf{v}^e\cdot\mathbf{n}^e\rangle_{pe\cap\Lambda_{h,D}}\} =$$
$$=\sum_{e\in\mathcal{E}_h}\{-\langle p_D^e,\mathbf{v}^e\cdot\mathbf{n}^e\rangle_{pe\cap\Lambda_D}+(z^e,\nabla\cdot\mathbf{v}^e)_{0,e}-\langle z^e,\mathbf{v}^e\cdot\mathbf{n}^e\rangle_{0,\partial e}\},$$
$$\sum_{e\in\mathcal{E}_h}-(\nabla\cdot\mathbf{u}^e,\phi^e)_{0,e}=\sum_{e\in\mathcal{E}_h}-(Q^e,\phi^e)_{0,e}$$
$$\sum_{e\in\mathcal{E}_h}\{\langle\mathbf{n}^e\cdot\mathbf{u}^e,\mu^e\rangle_{0,\Gamma_h}-\langle\sigma\lambda^e,\mu^e\rangle_{0,\partial e\cap\partial\Omega}\}=\sum_{e\in\mathcal{E}_h}\langle\mathbf{u}_N^e-\sigma\lambda_N^e,\mu^e\rangle_{0,\partial e\cap\partial\Omega}$$
$$(\mathbf{v},\phi,\mu)\in\mathbf{H}(div,\mathcal{E}_h)\times L_2(\Omega)\times H^{\frac{1}{2}}(\Gamma_h), \tag{5}$$

solved in appropriate functional spaces. We use a mixed-hybrid finite element method with lowest order Raviart-Thomas basis to discretize the system (5). This leads to a system of linear algebraic equations

$$\begin{array}{lllllll}\mathbb{A}\mathbf{u} & + & \mathbb{B}\mathbf{p} & + & \mathbb{C}\boldsymbol{\lambda} & = & \mathbf{q}_1\\ \mathbb{B}^T\mathbf{u} & & & & & = & \mathbf{q}_2\\ \mathbb{C}^T\mathbf{u} & + & & & \mathbb{F}\boldsymbol{\lambda} & = & \mathbf{q}_3.\end{array} \tag{6}$$

for unknowns **u** (velocity of flow expressed as an inter-element flux), **p** (modified pressure on the elements) and λ (modified pressure on the sides of elements). This system is symmetric and indefinite. We use a specialized iterative solver for solution of the system (6), based on the Schur complement method (see Mary, Rozložník, Tůma 2000).

An example of a real problem

Two holes, PTP3 and PTP4A, were drilled as a part of the grant project VaV 630/3/00. The drillholes are located in an extremely compact granitoid massif, near village Potůčky in Krušné Hory Mountains in the Czech Republic. Depth of the drillholes is 350 m (PTP3) and 300m (PTP4A); their horizontal distance is 10 m at the surface and approx. 15 m at the bottom of the drillhole PTP4A. Almost all available field measurements for achieving data about the fractured environment were performed. These included core scanning, acoustic television, electrical conductivity measurements, mineralogical analysis, chemical analysis, electron microscopy of the fracture surfaces and several others. The main result of these measurements is a description of fractures in the massif. Figure 2 shows an example of the results of the measurements.

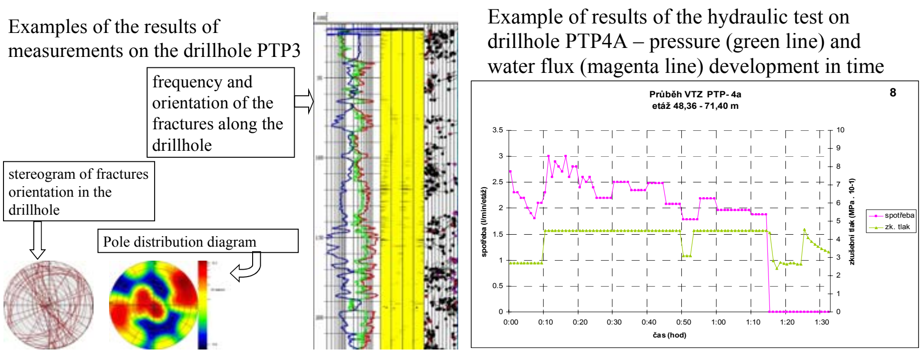

Figure 2. Results of the field measurements

The pumping and injection tests were also performed as well as tests of hydraulic communication between various parts of the drillholes. A tracer test was also conducted. Results of the tests gave us a suitable set of data for numerical modelling of the close neighborhood of the drillholes. At the beginning we tried to create a mesh consisting of the same number of fractures as observed in the drillhole. This attempt was not very successful because of a huge amount of generated fractures. When the computational domain had dimensions 20x30x20 m, and when we included fractures bigger than 0.20 m in diameter, we had a mesh with several tens of thousands of fractures. Calculation on such a large mesh is not possible due to computational costs caused by the excessive number of fractures and ill-conditioned state matrix of the linear equation system.

Therefore if was necessary to reduce the number of fractures. We have eliminated all fractures smaller than 2 m in diameter. There was proved that the importance of such fractures for the flow field is not very high. After this elimination there left only approx. 50 fractures in the domain. Calculation in this case was successful. Then we calibrated the model by the results of injection tests. After calibration, we achieved the difference between model and the reality better than 15%. Examples of the results are shown in Figure 3.

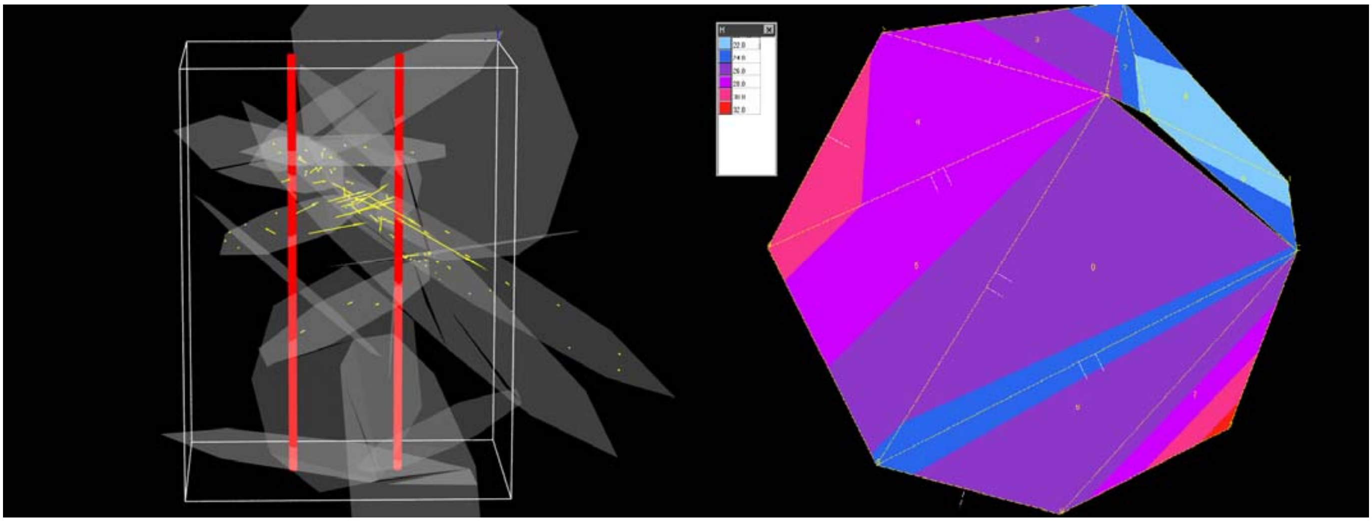

Figure 3. Results of the calculation of a real problem.

On the left of the Figure, the computational domain with drillholes is shown. Fractures are drawn as semi-transparent. Calculated vectors of velocity of flow are also shown. The right part shows pressure field and inter-element flux on one particular fracture of the mesh.

Conclusions

We have presented the most important information about our system for simulation flow in the fractured rock environment. The system is based on the discrete stochastic fracture network approach and uses mixed-hybrid FEM for approximation of the partial differential equations. It can be used for solution of hydrogeological problems of small scale. In near future we want to improve this limitation, and combine our approach with the equivalent porous medium approach.

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